# Cable Impedance Calculations with Parallel Circuits and Multi-Neutral Returns in Distribution Networks


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**Abstract-** This paper addresses impedance calculations in distribution networks. Two sets of paralleled, three-phase conductors with concentric neutrals and also a separate neutral are considered. An approximate method to calculate the impedance for this configuration is to calculate the impedance for one three-phase set of conductors, and then to divide this result by two. This paper compares this common practice with a more exact calculation that considers the mutual coupling between two sets of three-phase conductors. Errors in the current practice are shown to be significant. Comparisons are made for both phase and sequence impedances. Also, a common configuration of parallel conductors of different types is studied. It is shown that the approximate method may have up to 30% errors in the positive sequence impedance and power flows.



**Keywords**: impedance matrix, distribution networks, concentric neutral conductor, parallel circuits.


## I. INTRODUCTION

Distribution networks in many cities in the United States are becoming relatively old. Due to increased load growth and also the changing geographical location of load centers, a number of utilities in large cities are looking into re-design of these networks. As part of evaluating the re-design, power flow analysis is required. The results of the analysis depend upon the accuracy of the impedances used in the model.

In distribution networks as many as ten circuits may be run in parallel, where a circuit consists of three-conductors making up phases A, B, and C. Current practice is to approximate the impedance of n circuits in parallel (where all conductors are the same) by calculating the impedance of a single circuit and dividing the result by n.

This paper will consider two circuits in parallel. Consider the configuration shown in Fig. 1, where two sets of three-phase conductors with concentric neutrals are placed in two adjacent round ducts. Each set of three-phase conductor sets is referred to as a circuit. In addition, there is a separate neutral placed in a duct below the circuits. This is the configuration that will be analyzed in this paper.

The impedance for these circuits will be calculated using a common practice. Results from this calculation will be referred to as the Approximate method. More exact calculations of the impedance of the two circuits will then be performed. This method will be referred to as the Exact method. Error comparisons between the Approximate and Exact methods are made. Both calculations use the modified Carson's equations.

In deriving equivalent phase and sequence impedance matrices for both the Exact and Approximate methods, two different matrix reduction approaches are used, which are the Kron and the Neutral Return Current (NRC) reduction methods [1]. The Kron method should be used where the engineer expects significant earth return currents exist, such as where physical deterioration of the neutral has occurred. The Neutral Return Current method should be used where the engineer wishes to assume that all of the return current flows through the neutral conductors.

Figure 2 provides a schematic view of the conductors of Fig. 1. Figure 2 illustrates 13 current paths consisting of six phase conductors numbered 1-6, six concentric neutrals numbered 7-12, and the separate neutral numbered 13. As indicated in Fig. 2, at each manhole like phase conductors are tied together, such as conductor 1 and 4 are tied together. Also, at each manhole all neutrals, conductors 7-13, are tied together and grounded. Counting the earth return, there are actually 14 current paths.

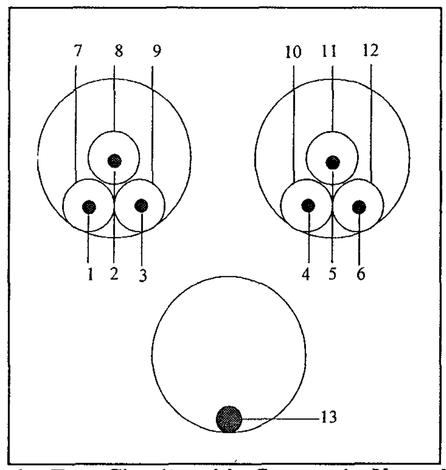


**Figure 1 Two Circuits with Concentric Neutral Cables in Separate Ducts with a Separate Grounded Neutral**

In the next section the Exact method applying the Modified Carson's equations to the impedance calculation is considered. This results in a 13x13 impedance matrix for the system illustrated in Fig. 1. The Kron and Neutral Return Current reduction methods are then used to reduce this matrix to equivalent 3x3 matrices. After the calculation using the Exact method, the common approximation is then used to obtain the impedance corresponding to Fig. 1. Section III presents the Approximate method and Section IV presents errors in the Approximate method.

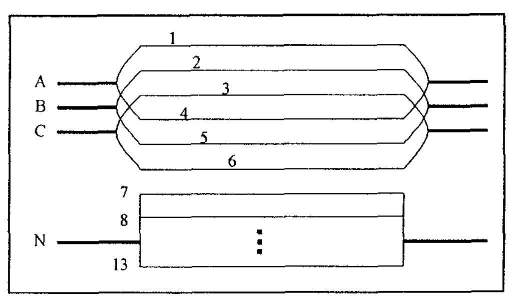


**Figure 2 Side View of Conductors Shown in Figure 1**

## II. EXACT CALCULATION

The Exact method is performed using the following steps:

Step 1: Use the Modified Carson's Equations to calculate the elements of the 13x13 impedance matrix, where the (i,j) impedance element multiplies current j to obtain the voltage drop in line i due to current j. [1,2]

Step 2: Reduce all neutral return currents to an equivalent neutral return current. Also reduce all like phase currents to an equivalent phase current. These reductions result in a 4x4 impedance matrix.

Step 3: Apply both Kron and Neutral Return Current reduction methods to the 4x4 impedance matrix obtained in Step 2 to obtain 3x3 equivalent phase impedance matrices.

Step 4: Transform the 3x3 phase impedance matrices obtained in Step 3 to sequence impedance matrices.

### *A. Primitive Impedance Matrix*

First consider the calculation of the self-impedance and mutual-impedance between any two conductors. Applying the Modified Carson's Equations we have [1]

$$Z_{ii} = \left(r_i + 0.0953\right) + j0.12134\left(\ln\frac{1}{GMR_i} + 7.93402\right) \quad (\Omega / mile) \qquad (1)$$

$$Z_{ij} = 0.0953 + j0.12134\left(\ln\frac{1}{D_{ij}} + 7.93402\right) \quad (\Omega / mile) \qquad (2)$$

where $r_i$ = conductor resistance

$GMR_i$ = the GMR for conductor i

$D_{ij}$ = spacing between conductors i and j.

Note that the earth return path has already been accounted for in (1) and (2).

There are 156 values of $D_{ij}$ to be determined for the 13x13 matrix. Referring to Fig. 3, $D_{ij}$ is calculated in one of four ways as follows:

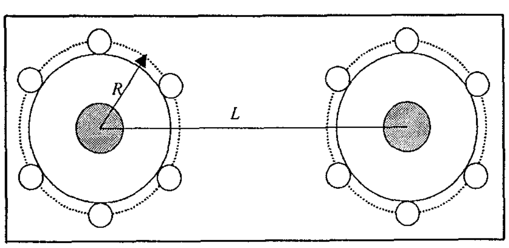


**Figure 3 Configuration of Two Phase Conductors with Concentric Neutrals, where c = 6**

(1) For a phase conductor to its corresponding concentric neutral, $D_{ij} = R$

(2) For a phase conductor to an adjacent phase conductor, $D_{ij} = L$

(3) For a phase conductor to an adjacent concentric neutral, $D_{ij} = \sqrt[c]{L^c - R^c}$

(4) For a concentric neutral to an adjacent concentric neutral, $D_{ij} = L$

where $R$ = radius of a circle going through the center of the concentric neutral strands

$L$ = center-to-center distance between phase conductors

$c$ = number of concentric neutral strands.

Equations (1) and (2) are used to calculate the elements of the 13x13 symmetric primitive impedance matrix. Inverting this matrix gives an admittance matrix, also symmetric, that relates the phase currents to the phase voltage drops as follows:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \\ I_{10} \\ I_{11} \\ I_{12} \\ I_{13} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} & Y_{16} & Y_{17} & Y_{18} & Y_{19} & Y_{1,10} & Y_{1,11} & Y_{1,12} & Y_{1,13} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} & Y_{26} & Y_{27} & Y_{28} & Y_{29} & Y_{2,10} & Y_{2,11} & Y_{2,12} & Y_{2,13} \\ & & & \bullet & & & \bullet & & & \bullet & & & \\ & & & \bullet & & & \bullet & & & \bullet & & & \\ & & & \bullet & & & \bullet & & & \bullet & & & \\ Y_{13,1} & Y_{13,2} & Y_{13,3} & Y_{13,4} & Y_{13,5} & Y_{13,6} & Y_{13,7} & Y_{13,8} & Y_{13,9} & Y_{13,10} & Y_{13,11} & Y_{13,12} & Y_{13,13} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \\ \Delta V_5 \\ \Delta V_6 \\ \Delta V_7 \\ \Delta V_8 \\ \Delta V_9 \\ \Delta V_{10} \\ \Delta V_{11} \\ \Delta V_{12} \\ \Delta V_{13} \end{bmatrix} \qquad (3)$$

### *B. Admittance Matrix Reduction: From 13x13 to 4x4*

Equation (3) will now be reduced to a 4x4 matrix equation. This will be performed by adding appropriate rows and by making use of voltage drops that are equal in paralleled conductors.

From Fig. 2 we have

$$I_a = I_1 + I_4 \qquad (4)$$

$$I_b = I_2 + I_5 \qquad (5)$$

$$I_c = I_3 + I_6 \qquad (6)$$

$$I_n = \sum_{k=7}^{13} I_k \qquad (7)$$

also,

$$\Delta V_a = \Delta V_1 = \Delta V_4 \qquad (8)$$

$$\Delta V_b = \Delta V_2 = \Delta V_5 \qquad (9)$$

$$\Delta V_c = \Delta V_3 = \Delta V_6 \qquad (10)$$

$$\Delta V_n = \Delta V_7 = \Delta V_8 = \text{L} = \Delta V_{13} \qquad (11)$$

Making use of (4) – (11) in (3), we have

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{aa} & Y_{ab} & Y_{ac} & Y_{an} \\ Y_{ba} & Y_{bb} & Y_{bc} & Y_{bn} \\ Y_{ca} & Y_{cb} & Y_{cc} & Y_{cn} \\ Y_{na} & Y_{nb} & Y_{nc} & Y_{nn} \end{bmatrix} \begin{bmatrix} \Delta V_a \\ \Delta V_b \\ \Delta V_c \\ \Delta V_n \end{bmatrix} \qquad (12)$$

where

$$Y_{aa} = Y_{11} + Y_{14} + Y_{41} + Y_{44}$$

$$Y_{ab} = Y_{ba} = Y_{12} + Y_{15} + Y_{42} + Y_{45}$$

$$Y_{ac} = Y_{ca} = Y_{13} + Y_{16} + Y_{43} + Y_{46}$$

$$Y_{bb} = Y_{22} + Y_{25} + Y_{52} + Y_{55}$$

$$Y_{bc} = Y_{cb} = Y_{23} + Y_{26} + Y_{53} + Y_{56}$$

$$Y_{cc} = Y_{33} + Y_{36} + Y_{63} + Y_{66}$$

$$Y_{an} = Y_{na} = \sum_{k=7}^{13} \left( Y_{1,k} + Y_{4,k} \right)$$

$$Y_{bn} = Y_{nb} = \sum_{k=7}^{13} \left( Y_{2,k} + Y_{5,k} \right)$$

$$Y_{cn} = Y_{nc} = \sum_{k=7}^{13} \left( Y_{3,k} + Y_{6,k} \right)$$

$$Y_{nn} = \sum_{k=7}^{13} \sum_{l=7}^{13} \left( Y_{k,l} \right)$$

Note that $Y_{ab} = Y_{ac} = Y_{ba} = Y_{bc} = Y_{ca} = Y_{cb}$ because the configuration in Fig. 1 is symmetric for all three phases.

### *C. Impedance Matrix Reduction: From 4x4 to 3x3*

Rewriting (12) in impedance form we have

$$\begin{bmatrix} \Delta V_a \\ \Delta V_b \\ \Delta V_c \\ \Delta V_n \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn} \\ Z_{na} & Z_{nb} & Z_{nc} & Z_{nn} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_n \end{bmatrix} \tag{13}$$

Also, $Z_{ab} = Z_{ac} = Z_{ba} = Z_{bc} = Z_{ca} = Z_{cb}$ .

Having the 4x4 symmetric impedance matrix shown in (13), we can now use two different matrix reduction methods to obtain an equivalent 3x3 phase impedance matrix. The two methods are the Kron reduction and the Neutral Return Current (NRC) reduction. The Kron reduction method assumes that the return current divides itself between the neutral conductors and earth, whereas the Neutral Return Current method assumes that all of the return current flows through the neutral conductors present [1]. Applying these assumptions results in the following matrix element transformation for Kron reduction

$$Z'_{ij} = Z_{ij} - \frac{Z_{in} \cdot Z_{jn}}{Z_{nn}} \tag{14}$$

where $Z'_{ij}$ = (i,j) element in equivalent 3x3 impedance matrix.

Similarly, the matrix element transformation for the Neutral Return Current method is given by

$$Z'_{ij} = Z_{ij} + Z_{nn} - Z_{in} - Z_{jn} \tag{15}$$

It should be noted that the above reductions only apply to wye connected systems. Both reductions result in a 3x3 equivalent phase impedance matrix which is symmetric, given by

$$Z'_p = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \tag{16}$$

Applying the symmetrical components transformation to (16) gives

$$Z'_s = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_+ & 0 \\ 0 & 0 & Z_- \end{bmatrix} \tag{17}$$

The notation used in (16) and (17) will be used in Section IV when results are presented and compared.

## III. APPROXIMATE METHOD

In calculating the impedance for the configuration shown in Fig. 1, the Approximate method assumes that there is no coupling between the two sets of conductors in the separate ducts. Hence, this practice only considers one set of conductors and the independent neutral return, i.e., conductors 1,2,3, 7, 8, 9, and 13 in Fig. 1. The steps used in the Approximate method calculation are:

Step 1: Use the Modified Carson's Equations to calculate the elements of the 7x7 impedance matrix, where the (i,j) impedance element multiplies current j to obtain the voltage drop in line i due to current j. [1,2]

Step 2: Reduce the neutral return currents of conductors 7, 8, 9, and 13 to an equivalent neutral return current. These reductions result in a 4x4 impedance matrix.

Step 3: Apply both Kron and Neutral Return Current reduction methods to the 4x4 impedance matrix obtained in Step 2 to obtain 3x3 equivalent phase impedance matrices.

Step 4: Transform the 3x3 phase impedance matrices obtained in Step 3 to sequence impedance matrices.

Step 5: The matrix of Step 4 is divided by 2 to get the final result.

This appears to be a good engineering approximation because the two circuits are in parallel. However, the next section shows that this approximation may result in significant impedance errors, compared with the Exact method presented in Section II.

**Table 1 Sequence Impedance Results with Kron Reduction**

| | Exact Method (Ω/1000ft) | | | | Approximate Method (Ω/1000ft) | | | | % Error | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | R0 | X0 | R+ | X+ | R0 | X0 | R+ | X+ | R0 | X0 | R+ | X+ |
| 350 MCM AA | 0.5829 | 0.1762 | 0.1752 | 0.0982 | 0.5409 | 0.2222 | 0.1752 | 0.0983 | -7.21 | 26.11 | 0.00 | 0.10 |
| 500 MCM AA | 0.4415 | 0.1288 | 0.1235 | 0.0943 | 0.4187 | 0.1608 | 0.1235 | 0.0942 | -5.16 | 24.84 | 0.00 | -0.11 |
| 750 MCM AA | 0.2941 | 0.0863 | 0.0865 | 0.0896 | 0.2857 | 0.1022 | 0.0865 | 0.0896 | -2.86 | 18.42 | 0.00 | 0.00 |
| 1000 MCM AA | 0.2344 | 0.0722 | 0.0697 | 0.0854 | 0.2293 | 0.0830 | 0.0697 | 0.0854 | -2.18 | 14.96 | 0.00 | 0.00 |

**Table 2 Sequence Impedance Results with Neutral Return Current (NRC) Reduction**

| | Exact Method (Ω/1000ft) | | | | Approximate Method (Ω/1000ft) | | | | % Error | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | R0 | X0 | R+ | X+ | R0 | X0 | R+ | X+ | R0 | X0 | R+ | X+ |
| 350 MCM AA | 0.5943 | 0.1420 | 0.1752 | 0.0982 | 0.5615 | 0.1825 | 0.1751 | 0.0982 | -5.52 | 28.52 | -0.06 | 0.00 |
| 500 MCM AA | 0.4469 | 0.1075 | 0.1235 | 0.0943 | 0.4287 | 0.1339 | 0.1236 | 0.0943 | -4.07 | 24.56 | 0.08 | 0.00 |
| 750 MCM AA | 0.2956 | 0.0767 | 0.0865 | 0.0896 | 0.2884 | 0.0893 | 0.0865 | 0.0896 | -2.44 | 16.43 | 0.00 | 0.00 |
| 1000 MCM AA | 0.2352 | 0.0658 | 0.0696 | 0.0853 | 0.2308 | 0.0742 | 0.0697 | 0.0853 | -1.87 | 12.77 | 0.14 | 0.00 |

**Table 3 Conductor Configuration**

| | Conductor Type | GMR of Phase Conductor (inch) | Resistance of Phase Conductor (Ω/mile) | Number of Concentric Neutral Strands | Resistance of Concentric Neutral (Ω/mile) | GMR of Concentric Neutral (inch) |
|---|---|---|---|---|---|---|
| 350 MCM AA | 1 | 0.262 | 0.342 | 14 | 0.9821 | 0.590 |
| 500 MCM AA | 2 | 0.312 | 0.242 | 12 | 0.7339 | 0.665 |
| 750 MCM AA | 3 | 0.385 | 0.159 | 12 | 0.4646 | 0.770 |
| 1000 MCM AA | 4 | 0.445 | 0.122 | 12 | 0.3696 | 0.845 |
| 4/0 CU | Independent Neutral | 0.200 | 0.311 | -- | -- | -- |

## IV. COMPARING APPROXIMATE AND EXACT METHOD

Considering the impedance of the conductor system shown in Fig. 1, results from the Approximate method presented in Section III are to be compared with the results from the Exact method presented in Section II. The notation of (16) and (17) will be used in the comparisons.

Tables 1-2 present the comparisons for four representative concentric neutral conductors with an independent neutral. Results where the Kron reduction approach is used are presented in Table 1. Table 2 shows results for the Neutral Return Current approach. Table 3 presents parameters for the conductors of tables 1-2. The last conductor in Table 3 is the independent neutral return used for each of the calculations [3]. The value shown in the Conductor Type column of Table 3 is used to identify the conductors.

From tables 1 and 2 it may be noted that there is essentially no error in the positive sequence quantities. This result could have been anticipated from the theory of balanced three-phase circuits.

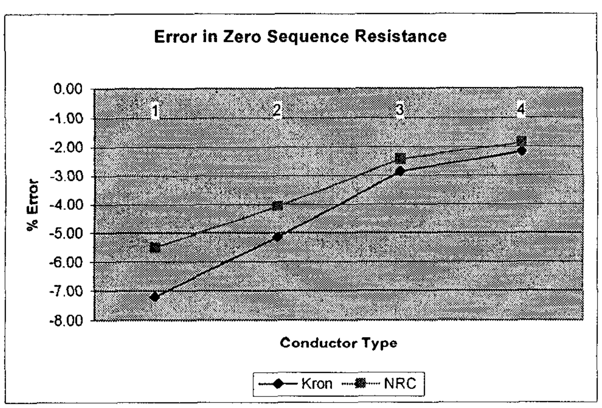


**Figure 4 Percentage Error of $R_0$ Approximate Calculation for Both Kron and Neutral Return Current (NRC) Reduction**

The errors in the zero sequence quantities shown in tables 1-2 are significant. The zero sequence reactance has the largest errors, ranging from approximately 15% for the largest conductor to almost 29% for the smallest conductor.

Figure 4 plots the percent error in $R_0$ and Fig. 5 plots the percent error in $X_0$ for both the Kron and Neutral Return Current methods, where the conductor type is shown in Table 3. In both cases larger conductors result in smaller errors. But even for the largest conductor, 1000 MCM AA, the errors are significant. $R_0$ is always approximated low, whereas $X_0$ is always approximated high.

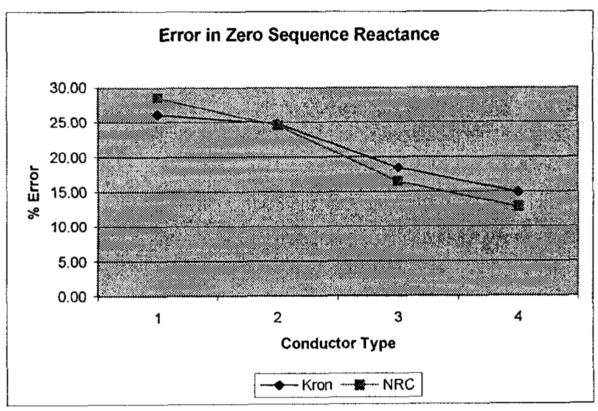


**Figure 5 Percentage Error of $X_0$ Approximate Calculation for Both Kron and Neutral Return Current (NRC) Reduction**

Figure 6 plots $R_0$ in ohms for the Exact method for both the Kron and Neutral Return Current reduction methods. Because the curves are so close together the plots appear to lie on top of one another in Fig. 6. Likewise, Fig. 7 plots $X_0$ in ohms for the Exact method for both the Kron and Neutral Return Current methods. This plot presents a range of values for $X_0$ which fall between the Kron assumptions and the Neutral Return Current assumptions.

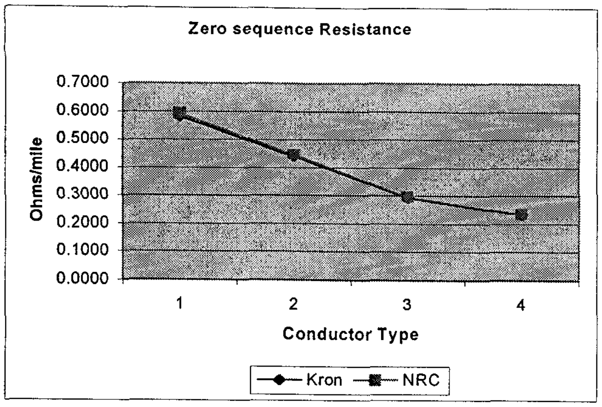


**Figure 6 Results for $R_0$ by Exact Method for Both Kron and Neutral Return Current (NRC) Reductions**

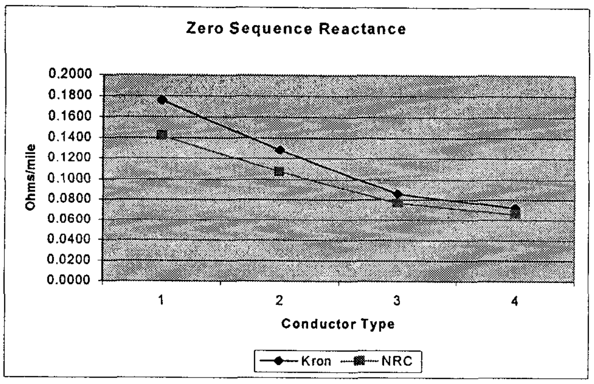


**Figure 7 Results for $X_0$ by Exact method for Both Kron and Neutral Return Current (NRC) Reductions**

## V. PARALLEL CONDUCTORS OF DIFFERENT TYPES

Different types of conductors may be placed in the same manhole. For instance, a set of three-phase 350 MCM CU conductors is placed in one duct, while a set of three-phase 500 MCM AA conductors is placed in a second duct. Also, a 4/0 Cu conductor as an independent neutral return is placed in a third duct. In this configuration, neglecting the mutual coupling causes considerable error. Calculation results for the impedances are shown in Table 4.

**Table 4 Sequence Impedance Calculated from Exact and Approximate Methods with Kron and NRC Reductions**

| | Exact Method (Ω/1000ft) | | | | Approximate Method (Ω/1000ft) | | | |
|---|---|---|---|---|---|---|---|---|
| | R0 | X0 | R+ | X+ | R0 | X0 | R+ | X+ |
| Kron | 0.3920 | 0.1303 | 0.0718 | 0.0961 | 0.4389 | 0.1874 | 0.1092 | 0.1057 |
| NRC | 0.3954 | 0.1085 | 0.0718 | 0.0961 | 0.4512 | 0.1547 | 0.1092 | 0.1057 |

In the above table, the Exact method is to calculate the full impedance matrix and then crunch it to 3x3 with Kron or NRC reduction. The Approximate method is to calculate the impedance for two conductors separately, and then calculate the overall impedance by considering them as parallel impedances.

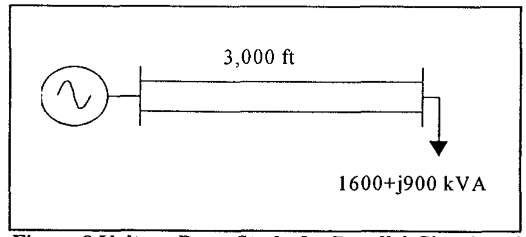


**Figure 8 Voltage Drop Study for Parallel Circuits with Different Cable Types**

Now let us consider voltage drops predicted by impedances of Table 4 with the Exact and Approximate methods with Kron reduction. The system to be considered is shown in Fig. 8. The feeder is 3,000 feet long and a customer load of 1600kW+900kVAR for each phase is connected at the end point of the feeder. The bus voltage at the source end is 120V.

The results of the load flow are shown in Table 5. The voltage at the end point of the feeder is 109.6V with the actual impedance, while it is 105.6V with the approximate impedance. That is, the voltage drops are 10.4V and 14.4V, respectively. The voltage drop error in this case is 38.6%.

**Table 5 Power Flow Results for a Feeder in Figure 8**

| | S (kVA) | I (A) | V (V) | ΔV (V) |
|---|---|---|---|---|
| Exact | 1600+j900 | 725 | 109.6 | 10.4 |
| Approx. | 1600+j900 | 753 | 105.6 | 14.4 |

The above results show that the Exact method allows the feeder to carry heavier loads than that predicted by the Approximate method. That is, the capacity of the system is larger with the Exact method. Another aspect to be considered here is that the Approximate method may predict a voltage that is out of limits, where the voltage may actually be within limits.

## VI. CONCLUSIONS

This paper has compared approximate impedance calculations used by utilities in distribution networks with more exact calculations. Both errors in impedances and voltage drops have been shown to be significant with the Approximate method. From the comparisons the following observations may be made:

- The Approximate method is satisfactory for balanced power flow studies, where all conductors involved are of the same type.
- The Approximate method would result in significant errors in unbalanced power flow studies.
- The Approximate method would result in significant errors in fault current studies.
- Impedance errors become smaller as the conductors become larger, but even with larger conductors, the errors are significant.
- A combination of different types of conductors may cause errors in the positive sequence impedance with the Approximate method. Hence, errors may be produced even in balanced power flow studies.
- Voltage drop errors with the Approximate method may be unacceptable.
- The Approximate method may under predict system capacity.

This work has shown that more accurate impedance models and power flow calculations are needed in complex, networked systems.

## VII. REFERENCE

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## VIII. BIOGRAPHIES

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